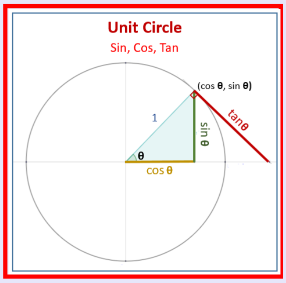
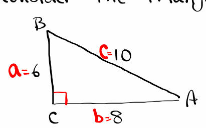
Math 241

Winter 2023

Lecture 6



Consider the triange ABC below



1) Verify that $\triangle ABC$ is a right triangle.

$a^2+b^2=c^2$

$6^2+8^2=10^2$

$36+64=100$

$100=100$ ✓

2) Find its perimeter & area.

$P=a+b+c=6+8+10=24$ units

$\text{Area}=\frac{bh}{2}=\frac{8\cdot 6}{2}=\frac{48}{2}=24$ units^2

3) Find its area using Heron's Formula

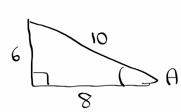
$\text{Area}=\sqrt{S(S-a)(S-b)(S-c)}$

$S=\frac{a+b+c}{2}$

$S=\frac{P}{2}=\frac{24}{2}=12$

$\text{Area}=\sqrt{12(12-6)(12-8)(12-10)}=\sqrt{12\cdot 6\cdot 4\cdot 2}=\sqrt{576}=\boxed{24}$

4) Find the measure of angle A.



$\sin A=\frac{6}{10}=.6 \Rightarrow A=\sin^{-1}(.6)\approx 37^\circ$

"Sine inverse"

$\cos A=\frac{8}{10}=.8 \Rightarrow A=\cos^{-1}(.8)\approx 37^\circ$

"Cosine inverse"

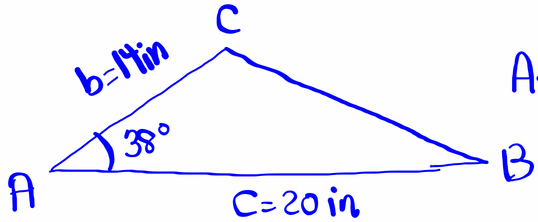
$\tan A=\frac{6}{8}=\frac{3}{4}=.75 \Rightarrow A=\tan^{-1}(.75)\approx 37^\circ$

"tangent inverse"

Find area of triangle ABC such that

$b = 14$ in, $C = 20$ in, and $A = 38°$. Drawing Required.

We have SAS.



$$\text{Area} = \frac{1}{2} bc \sin A$$

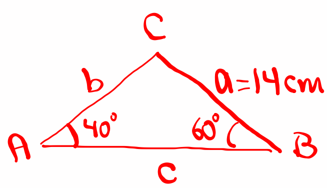
$$= \frac{1}{2} \cdot 14 \cdot 20 \cdot \sin 38°$$

$$= 86.1926\ldots$$

$$\text{Area} \approx 86 \text{ in}^2$$

Solve triangle ABC with $\measuredangle A = 40°$, $\measuredangle B = 60°$,

and $a = 14$ cm.



$$A + B + C = 180°$$

$$40° + 60° + C = 180° \Rightarrow \boxed{C = 80°}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Sines

$$\frac{\sin 40°}{14} = \frac{\sin 60°}{b} = \frac{\sin 80°}{c}$$

$$\frac{\sin 40°}{14} = \frac{\sin 60°}{b} \Rightarrow b \sin 40° = 14 \sin 60° \Rightarrow b = \frac{14 \sin 60°}{\sin 40°}$$

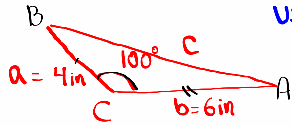
$$\boxed{b \approx 19 \text{ cm}}$$

$$\frac{\sin 40°}{14} = \frac{\sin 80°}{c} \Rightarrow c = \frac{14 \sin 80°}{\sin 40°} \qquad \boxed{c \approx 21 \text{ cm}}$$

Solve $\triangle ABC$ such that $a = 4$ in, $b = 6$ in, and $C = 100°$.

We have SAS

Use Law of Cosines



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 4^2 + 6^2 - 2(4)(6)\cos 100°$$
$$= 60.335 \Rightarrow c = \sqrt{60.335}$$
$$c = 7.768$$
$$\boxed{c \approx 8 \text{ in}}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{4} = \frac{\sin B}{6} = \frac{\sin 100°}{8}$$

$$\frac{\sin A}{4} = \frac{\sin 100°}{8} \qquad 8\sin A = 4\sin 100° \qquad \sin A = \frac{4\sin 100°}{8}$$

$$\sin A = \frac{\sin 100°}{2} \Rightarrow \sin A = .492 \Rightarrow A = \sin^{-1}(.492)$$
$$\boxed{A \approx 29°} \quad *$$

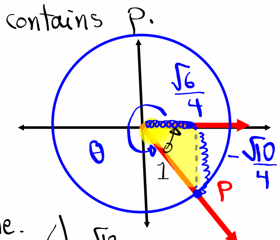
$A + B + C = 180°$

$29° + B + 100° = 180° \rightarrow \boxed{B = 51°}$

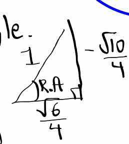
Consider the point $P\left(\frac{\sqrt{6}}{4}, -\frac{\sqrt{10}}{4}\right)$ and angle $\theta > 0$
$x > 0$ $\quad y < 0$
such that its terminal side contains P.

1) Draw angle θ.

$$x^2 + y^2 = r^2$$
$$\left(\frac{\sqrt{6}}{4}\right)^2 + \left(-\frac{\sqrt{10}}{4}\right)^2 = \frac{6}{16} + \frac{10}{16} = \frac{16}{16} = 1$$



2) Draw its reference triangle.



3) Complete the chart below

$\sin\theta = -\frac{\sqrt{10}}{4}$ $\qquad$ $\csc\theta = -\frac{4}{\sqrt{10}}$

$\cos\theta = \frac{\sqrt{6}}{4}$ $\qquad$ $\sec\theta = \frac{4}{\sqrt{6}}$

$\tan\theta = -\frac{\sqrt{15}}{3}$ $\qquad$ $\cot\theta = -\frac{3}{\sqrt{15}}$

Need to be rationalized

$$\frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{10}}{\cancel{4}}}{\frac{\sqrt{6}}{\cancel{4}}} = -\frac{\sqrt{10}}{\sqrt{6}} = -\frac{\cancel{\sqrt{2}}\sqrt{5}}{\cancel{\sqrt{2}}\sqrt{3}} = -\frac{\sqrt{15}}{3}$$

Convert 40° to radians. Give exact Ans.

$180° = \pi$

$1° = \frac{\pi}{180}$

$40° = \frac{\overset{2}{\cancel{40}}\pi}{\underset{9}{\cancel{180}}}$

$40° = \frac{2\pi}{9}$

Convert $\frac{3\pi}{5}$ to degrees.

$\frac{3\pi}{5} = \frac{3(180°)}{5} = \boxed{108°}$

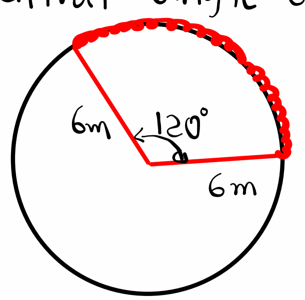
Find complement of $\alpha = 72°$.

Comp. $= 90° - 72° = \boxed{18°}$

Find supplement of $\alpha = \frac{3\pi}{7}$.

Suppl. $= \pi - \frac{3\pi}{7} = \frac{7\pi}{7} - \frac{3\pi}{7} = \boxed{\frac{4\pi}{7}}$

1) Draw a Sector with central angle of 120° and radius 6 m.



2) Find its arc length.

$S = r\theta = 6 \cdot \frac{2\pi}{3} = \boxed{4\pi \text{ m}}$

$\hookrightarrow 120° = 2(60°) = 2 \cdot \frac{\pi}{3}$

3) Find its area.

$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \cdot 6^2 \cdot \frac{2\pi}{3} = \boxed{12\pi \text{ m}^2}$

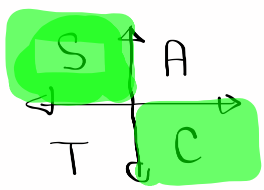
$\tan\alpha = -\dfrac{\sqrt{5}}{10}$

1) Find $\tan(-\alpha)$

$\tan(-\alpha) = -\tan\alpha = -\left(\dfrac{-\sqrt{5}}{10}\right) = \dfrac{\sqrt{5}}{10}$

2) Find $\cot\alpha = -\dfrac{10}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = -\dfrac{10\sqrt{5}}{5} = \boxed{-2\sqrt{5}}$

3) Discuss what quadrants can α belong to.



$\tan\alpha = -\# < 0$

QII or QIV

Simplify $\dfrac{\sec\theta \cdot \cot\theta}{\csc\theta} = \dfrac{\dfrac{1}{\cancel{\cos\theta}} \cdot \dfrac{\cancel{\cos\theta}}{\sin\theta}}{\dfrac{1}{\sin\theta}}$

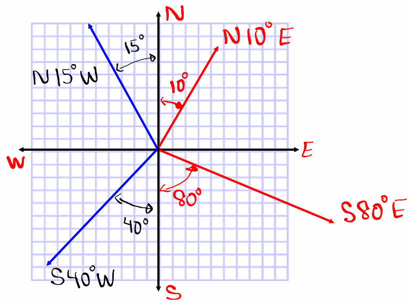
$= \dfrac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\sin\theta}} = \boxed{1}$

Verify $\dfrac{\cot\theta}{\csc\theta} = \cos\theta$ ✓

$\dfrac{\cot\theta}{\csc\theta} = \dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}} = \dfrac{\cancel{\sin\theta} \cdot \dfrac{\cos\theta}{\cancel{\sin\theta}}}{\cancel{\sin\theta} \cdot \dfrac{1}{\cancel{\sin\theta}}} = \dfrac{\cos\theta}{1} = \cos\theta$

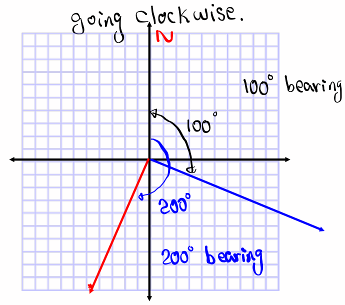
Bearing:

Method I: It is an acute angle ($<90°$) that the path makes with North or South lines.

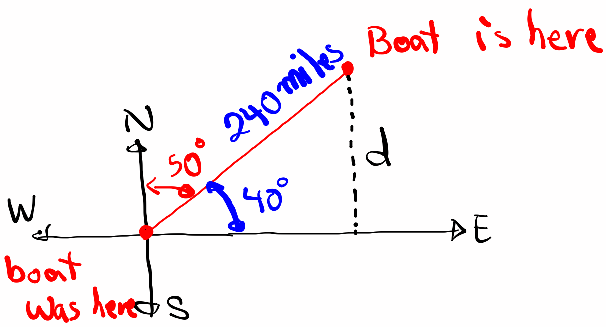


Method II: It is an angle measuring from North line going clockwise.

Positive



A boat is traveling on a bearing $\boxed{N\,50°\,E}$ for 240 miles. How many miles is the boat from east line at this time? Drawing Required



$$\sin 40° = \frac{d}{240}$$

$$d = 240 \sin 40°$$

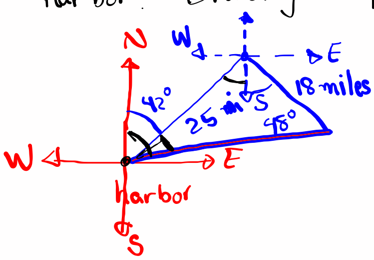
$$= 154.269$$

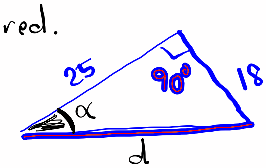
$$\approx \boxed{154 \text{ miles}}$$

A boat travels 25 miles in the direction of N 42° E.
then it goes 18 miles in the direction of S 48° E.

1) How far is the boat now from the harbor?

2) what is the bearing of the boat now from the harbor? Drawing required.





$d^2 = 25^2 + 18^2$

$= 949 \quad d \approx 30.805...$

$\boxed{d \approx 31 \text{ miles}}$

N 78° E

$\tan\alpha = \frac{18}{25}$

$\tan\alpha = .72 \quad \alpha = \tan^{-1}(.72)$

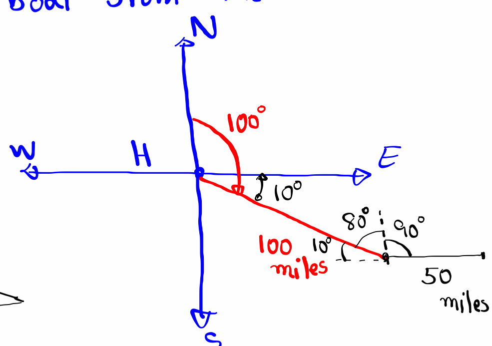
$42° + 36° = 78°$

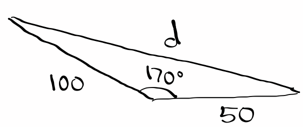
$\alpha \approx 36°$

A boat travels for 100 miles in the bearing of 100°. Then it goes 50 miles due east.

How far is the boat from the harbor.

Drawing required.





SAS ⇒ Law of Cosines

$d^2 = 100^2 + 50^2 - 2(100)(50)\cos 170°$

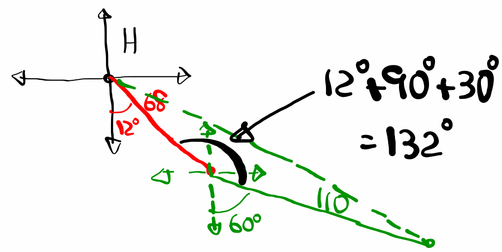
$= 22348.07753 \quad d = \sqrt{22348.07753} \quad d = 149.492...$

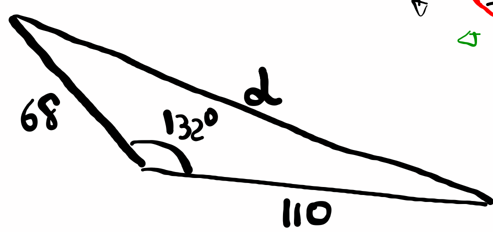
$d \approx \boxed{149 \text{ miles}}$

A ship travels S12°E for 68 miles.

It changes direction and travels S 60°E for 110 miles. How far is the ship from the harbor?

Drawing required.





SAS ⇒ Law of Cosines

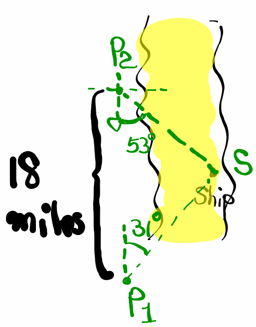
$d^2 = 68^2 + 110^2 - 2(68)(110)\cdot\cos 132°$

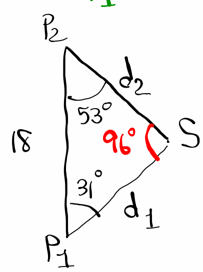
$\boxed{d \approx 164 \text{ miles}}$

Consider a river that runs North and South.

A ship is anchored off a straight shoreline.

From two observation points 18 miles apart on the shore, bearings of the ship are N 31°E and S 53°E. Find distance from the ship to both observation points.





$\boxed{\dfrac{\sin 53°}{d_1}} = \dfrac{\sin 31°}{d_2} = \boxed{\dfrac{\sin 96°}{18}}$

$\dfrac{\sin 53°}{d_1} = \dfrac{\sin 96°}{18} \Rightarrow d_1 = \dfrac{18 \sin 53°}{\sin 96°}$

do similar way ⇒ $\boxed{d_2 \approx \quad \text{miles}}$ $\quad d_1 \approx 14 \text{ miles}$

From Algebra

If you drive 60 mph for 4 hrs,

60 mi	60 mi	60 mi	60 mi
First hr	Second hr	3rd hr	4th hr

Total distance $4(60) = 240$ miles

$$\text{Linear Speed} = \frac{\text{How far you go}}{\text{How long it takes}}$$

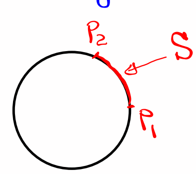
$$r = \frac{d}{t} \Rightarrow d = r \cdot t$$

Rate

Suppose an object moves along a circle, we need to find linear velocity

$$\underbrace{\text{Linear Velocity}} = \frac{\text{How far along the circle}}{\text{How long it takes}}$$

$$v = \frac{s}{t}$$



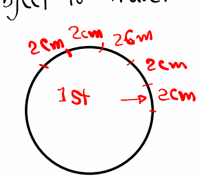
Ex: It takes 20 mins for an object to travel 5cm. on a circle

Linear Velocity $v = \frac{s}{t} = \frac{5\text{ cm}}{20\text{ min}}$

$= .25$ cm/min.

Find a linear velocity for an object to travel 10cm along a circle in 5 min.

$$v = \frac{s}{t} = \frac{10\text{cm}}{5\text{ min.}} = 2\text{ cm/min.}$$



$$\underbrace{\text{Angular Velocity}}_{\omega}$$

$$\omega = \frac{\theta}{t}$$

omega

$$\text{Angular velocity} = \frac{\text{How much angle}}{\text{How long it take}}$$

Angle must be in radians

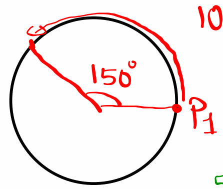
Ex: Point P travels $\frac{3\pi}{4}$ in 3 minutes.

Find angular velocity

$$\omega = \frac{\theta}{t} = \frac{\frac{3\pi}{4}}{3} = \pi/4 \text{ Rad./min.}$$

Point P travels 150° in 10 Seconds.

Find its angular velocity.



10 Seconds

$$\omega = \frac{\theta}{t} = \frac{5\pi/6}{10}$$

$$= \frac{5\pi}{6} \cdot \frac{1}{10} = \frac{\pi}{12} \text{ Rad./Sec.}$$

$$150^\circ = 150 \cdot 1^\circ = \overset{5}{\cancel{150}} \cdot \frac{\pi}{\underset{6}{\cancel{180}}}$$

What about Per min?

$$\frac{\pi \text{ Rad.}}{\cancel{12}\ \cancel{\text{Sec.}}} \cdot \frac{\overset{5}{\cancel{60}}\ \cancel{\text{Sec}}}{1 \text{ Min.}} = 5\pi \text{ rad/min.}$$

What about Per hour?

$$\frac{5\pi \text{ Rad}}{1\ \cancel{\text{Min.}}} \cdot \frac{60\ \cancel{\text{min.}}}{1 \text{ hr}} = 300\pi \text{ rad/hr}$$

Find linear velocity of a point moving along circular motion if it travels 6 cm in 2 seconds.

$$v = \frac{s}{t} = \frac{6\text{cm}}{2\text{ sec.}} = 3\text{ cm/sec.}$$

what about m/sec.?

$$\frac{3\,\cancel{\text{cm}}}{1\text{ Sec.}} \cdot \frac{1\text{ m}}{100\,\cancel{\text{cm}}} = \frac{3\text{m}}{100\text{ Sec.}}$$

what about m/min.?

$$\frac{3\text{ m}}{\cancel{100}^{5}\,\cancel{\text{Sec.}}} \cdot \frac{\cancel{60}^{3}\,\cancel{\text{Sec.}}}{1\text{ min.}} = \frac{9\text{ meters}}{5\text{ minutes}}$$

Find the distance covered with $v = 75$ mph in 15 Seconds.

$$v = \frac{s}{t} \Rightarrow s = vt$$

15 Seconds =

$$15\text{ Seconds} \cdot \frac{1\text{ hr}}{3600\text{ Seconds}}$$

$$= 75 \cdot \frac{15}{3600}$$

$$= \boxed{.3125\text{ miles}}$$

Find $\underbrace{\text{angular velocity}}$ for $\theta = 24\pi$ in 2.4 hrs.

$$\omega = \frac{\theta}{t} = \frac{24\pi}{2.4} = 10\pi \text{ Rad/hr}$$

Find θ if $\omega = 10$ Rad/sec, $r = 6$ft, $t = 2$min.

$t = 120$ sec.

$$\omega = \frac{\theta}{t} \Rightarrow \theta = \omega \cdot t$$

$$= 10 \cdot 120 = 1200 \text{ Radians.}$$

Find S $\quad S = r\theta = 6 \cdot 1200 = \boxed{7200 \text{ ft}}$

✓ $S = r\theta$ → Divide by r

✓ $V = \dfrac{S}{t}$

$$\frac{S}{r} = \frac{r\theta}{r}$$

✓ $\omega = \dfrac{\theta}{t}$ ← $\dfrac{S}{r} = \theta$

$$\omega = \frac{\frac{S}{r}}{t} = \frac{S}{rt} = \frac{1}{r} \cdot \frac{S}{t} = \frac{1}{r} \cdot V$$

$$\boxed{\omega = \frac{V}{r}} \Leftrightarrow \boxed{V = r\omega}$$

find v if $r = 2$ inches, and $\omega = 5$ rad./sec.

$v = r \cdot \omega$

$= 2 \cdot 5 = 10$ in/sec.

find ω when $r = 3$ cm and $v = 15$ cm/sec.

$\omega = \frac{v}{r} = \frac{15}{3} = 5$ Rad./sec.